

**tomSym** is a TOMLAB class for modeling optimization, constraint programming and optimal control problems in MATLAB. The modeling class is included with the general TOMLAB Base Module.

The class allows for rapid prototyping and modeling of a wide variety of problem types, including:

- Linear and quadratic programming
- Nonlinear and semidefinite programming
- Least squares problem (L1, L2, Infinity norm)
- Mixed-integer programming
- Multi-index variable optimization problems

The internal engine (parser) provides for a number of advantages:

- Analytical derivatives (1st and 2nd order) and sparsity patterns are automatically generated
- Possible to generate Matlab code for further processing
- Optimal modeling of expressions such as min, max, abs and more
- Compatible with MAD for automatic differentiation
- Support for if, then, else statements
- Ability to analyze pcoded MATLAB files
- Automated code simplifications and much more

## Code Generation

The main strength of tomSym is its ability to automatically and quickly compute symbolic derivatives for matrix expressions. The derivatives can then be converted into efficient Matlab code. The box contains a simple example that illustrates the use of tomSym.

In the example, the 2x1 symbol  $f$  is differentiated with respect to the 2x3 symbol  $A$ . The result is a 2x6 matrix, representing  $d(\text{vec}(f))/d(\text{vec}(A))$ .

```
>> toms y
>> toms 3x1 x
>> toms 2x3 A

>> f = (A*x).^(2*y)
f = tomSym(2x1):
(A*x).^(2*y)

>> derivative(f,A)

ans = tomSym(2x6):
(2*y)*setdiag((A*x).^(2*y-1))*kron(x',eye(2))
```

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## MATLAB Ezsolve Engine

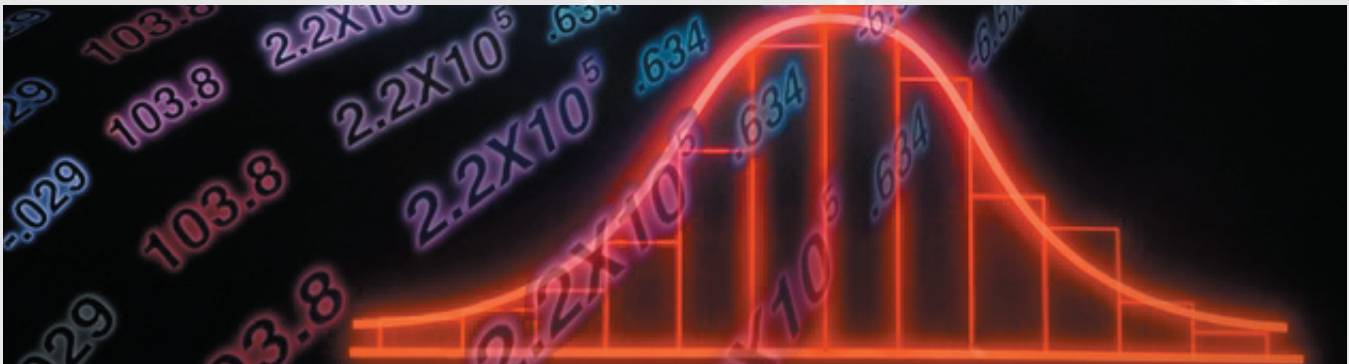
tomSym includes an **ezsolve** function, which needs minimal input to solve an optimization problem: only the objective function and the constraints.

Ezsolve determines the problem type, finds an appropriate solver, generates the problem, calls the optimization solver and returns the solution - all in one simple step.

```
toms integer x
toms y

objective = -6*x + 2*x^2 + 2*y^2 - 2*x*y;
constraints = {x+y<=1.9, x>=0, y>=0};

solution = ezsolve(objective,constraints);
```



## Features

- An interface for **bilinear semidefinite** problems has been implemented. It is also possible to solve nonlinear problems involving semidefinite constraints, using any nonlinear solver.
- If a user function is incompatible with tomSym, it can still be used in the symbolic computations, by giving it a **wrapper** for either automatic or numerical differentiation.
- MATLAB **code optimization** - The code generation function detects sub-expressions that occur more than once, and optimizes by creating temporary variables for those. tomSym also automatically implements algebraic simplifications of expressions.
- tomSym has the ability to **analyze pcode** for possible performance improvements and validation. Assume that we have a pcoded file with the following hidden content:

```
function y = pcodetest(a,b)
y = sin(a)+b.^2-30;
```

Then execute the following in MATLAB:

```
>> toms 3x1 a b
>> pcodetest(a,b)
ans = tomSym(3,1):
    -30+(sin(a)+b.^2)
```

Generated code

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## Examples

There are several examples included with tomSym for all areas of optimization. Linear, quadratic, conic, integer and norm optimization problems are a few of them. The tomSym model library contains more than 85 applied models for a variety of application areas, including: transportation, scheduling problems, financial optimization and chemical engineering.

### Linear Programming

The following code can be used to solve a LP (Linear Programming) problem:

```
toms 2x1 x
solution = ezsolve(c'*x, {A*x<=b_U, x_L<=x});
```

### Mixed-Integer Programming

Binary and integer restricted linear programming problems are easily modeled by defining tomSym variables that are integers and setting optional bounds.

The tomSym system allows for the conversion of models to the standard TOMLAB problem structure as shown in the example.

```
toms 28x1 integer x

objective = -c'*x;
constraints = {A*x<=b_U, 0<=x<=1};
Prob = sym2prob('mip',objective,constraints);
Result = tomRun('cplex', Prob, 1);
```

### Quadratic (Integer) Programming

The tomSym ezsolve engine will automatically detect that the problem is a convex (MI)QP problem and assign the most suitable solver:

```
toms integer x
toms y

objective = -6*x + 2*x^2 + 2*y^2 - 2*x*y;
constraints = {x+y<=1.9, x>=0, y>=0};

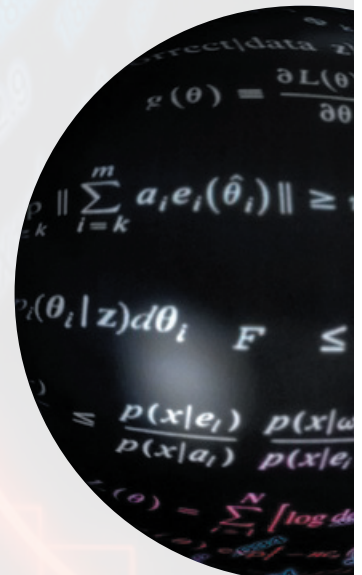
solution = ezsolve(objective,constraints);
```

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## SDP (Semi-Definite Programming)

SDP problems can be solved with PENSDP or PENBMI in TOMLAB. tomSym can also convert the problem to a nonlinear programming problem (NLP) by setting options.type = 'con' before calling ezsolve.

```
toms rab rac rbc
f = rac;
M = [1 rab rac; rab 1 rbc; rac rbc 1]
con = { positiveSemidefinite(M), ...
      -0.2 <= rab <= -0.1, ...
      0.4 <= rbc <= 0.5};

x0 = struct('rab', -0.15, 'rbc', 0.45, 'rac', 0);

option = struct;
options.type = 'sdp';
solution = ezsolve(f, con, x0, options);
```

```
toms x1 x2
alpha = 100;

% Objective function
f = alpha*(x2-x1^2)^2 + (1-x1)^2;

% Constraints
c = -x1^2 - x2;
con = {-1000 <= c <= 0
      -10 <= [x1;x2] <= 2};

% Initial conditions
x0 = {x1 == -1.2; x2 == 1};

% Compile and solve problem
solution = ezsolve(f, con, x0);
```

## Nonlinear Programming

In general there is nothing different with nonlinear models in tomSym. They are modeled just like a linear or quadratic problem. The modeling engine takes care of the rest, including first and second order derivatives and problem sparsity patterns.

## Least Squares

Least square problems can be modeled like any other problem with tomSym. The following is an L2-norm solution:

```
% Define the variable
toms 6x1 x_hat

% Define the residuals
residuals = y-a*x_hat;

%The L2 problem is solved
solution = ezsolve(residuals'*residuals, []);
```

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