Optimal Scheduling of Cogeneration Plants[1]

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Abstract

A cogeneration plant, feeding its output water into a district-heating grid, may include several types of energy producing units. The most important being the cogeneration unit, which produces both heat and electricity. Most plants also have a heat water storage. Finding the optimal production of both heat and electricity and the optimal use of the storage is a difficult optimization problem. This paper formulates a general approach for the mathematical modeling of a cogeneration plant. The model objective function is nonlinear, with nonlinear constraints. Internal plant temperatures, mass flows, storage losses, minimal up and down times and time depending start-up costs are considered. The unit commitment, i.e. the units on and off modes, is found with an algorithm based on Lagrangian relaxation. The dual search direction is given by the subgradient method and the step length by the Polyak rule II. The economic dispatch problem, i.e. the problem of determining the units production given the on and off modes, is solved using a combination of dynamic programming and general-purpose solvers. The model and algorithms are implemented in MATLAB.

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Unit	Name
HWC	Heat water central unit
ELH	Electric heater
HEP	Heat exchange pump
CHP	Cogeneration unit
HWS	Heat water storage

Table 1: Production units.

1 Introduction

In Sweden the district-heating technique is common for house warming, see [FW93]. In a district-heating system usually the heating units are gathered in a plant. The production plant, named cogeneration plant when there are production of both heat and electric power, may include several production units of different type, see Table 1. Together these units shall cover the required heat demand. The most dominating types of units are combustion units. These can either be *Heat water central units*, which are units for heat production only, or *Cogeneration units*, which produces both heat and electric power. Two other common types of units are *Heat exchange pumps* and *Electric heaters*. Normally production plants also include a *Heat water storage,* in which it is possible to store energy, i.e. hot water or steam, for later use.

When the electricity price is high the aim is to produce as much electric power as possible and make a profit of it at the electricity market. Using a cogeneration plant it is possible to have the process to operate with a much higher level of total cost effectiveness than a pure electric power plant. However, at this high degree of efficiency the produced heat and electric power are proportional, i.e. producing more electric power necessarily means producing more heat power. This proportionality is often a problem. The demand of electricity is higher during the working day, leading to a higher price in daytime. The heat demand on the other hand is depending on the out door temperature, i.e. it is lower in summer and higher in winter. This problem is partly solved using the Heat water storage. Discharging the storage when the electricity price is low, it is possible to charge it when the price is high, i.e. produce more heat power than the heat demand. Producing more heat power, that gives extra production of electric power that may be sold for a high price.

The Swedish district-heating systems are built for a forward water temperature of $80-120$ $°C$. The forward temperature is chosen to give a return temperature about $50-60$ $°C$. The input and output temperature of an individual unit does not have these restrictions, as the output from the units are mixed to give the correct forward temperature. The input temperature will be much higher than the return temperature when the units are connected in series. In all cases a high input temperature to a unit leads to a lower degree of efficiency. Previously this point has been neglected in the modeling of district-heating plants.

This paper considers the problem of finding the best production schedule for the near future using a combination of mathematical models and computer algorithms. The problem is called the short-term production-planning problem. All plant flows and internal plant temperatures are included in the model. This will lead to a better view of the pros and cons of the use of units in series, like Heat exchange pumps feeding its output water with relatively high temperature into a Cogeneration unit. The aim is to find the production that minimizes a mathematical model description of the cost to run the cogeneration plant taking into account different constraints on the production. The schedule is normally computed for the next twenty-four hour period. The intention is that the computations should be easy and fast and enable recomputation of the schedule as often as necessary. In principle each time new information arrives, about once an hour. This paper presents only the basic strategy used to model and solve the problem. A detailed description, including a literature review, a MATLAB implementation, numerical examples and algorithm performance analysis, is to be presented in the forthcoming licentiate thesis by Erik Dotzauer [Dot97].

2 Problem Formulation

When creating the mathematical model it is natural to partition the time horizon over which the problem is to be solved into a finite number of time intervals. Therefore, define I time intervals in the range $[0, T]$ defined by $I+1$ time points $\tau_0, \tau_1, ..., \tau_I$, where $\tau_0 = 0$ and $\tau_I = T$. The length of the ith time interval is $\Delta\tau_i = \tau_i - \tau_{i-1}$.

For unit k in time interval i, define the heat power production $q_{i,k}$, the electric power production $p_{i,k}$, the output temperature $t^f_{i,k}$, the input temperature $t^r_{i,k}$ and the mass flow rate $m_{i,k}$. Let $u_{i,k}$ be a binary variable indicating if unit k in time interval i is on or off, i.e. producing or not producing heat and electric power. If the unit is on $u_{i,k}$ is equal to one. If the unit is off $u_{i,k}$ is zero.

In every time interval the relation

$$q_{i,k} = c_p m_{i,k}(t^f_{i,k} - t^r_{i,k}) \tag{1}$$

holds, together with the upper and lower bounds

$$\underline{p}_{i,k} u_{i,k} \leq p_{i,k} \leq \overline{p}_{i,k} u_{i,k}, \tag{2}$$

$$\underline{t}^f_{i,k} u_{i,k} \leq t^f_{i,k} \leq \overline{t}^f_{i,k} u_{i,k} \tag{3}$$

and

$$\underline{m}_{i,k} u_{i,k} \leq m_{i,k} \leq \overline{m}_{i,k} u_{i,k}, \tag{4}$$

where c_p is defined as the specific heat capacity. The production of electricity, $p_{i,k}$, is always zero for a heat only unit.

Define $T_{i,k}$ to be the running time for unit k at the end of time interval i, i.e. the time since the unit was switched on. $T_{i,k}$ is negative if the unit has been off, positive otherwise. Let the equations describing $T_{i,k}$ be

$$T_{i,k} = \begin{cases} T_{i-1,k} + \Delta\tau_i & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 1 \\ \Delta\tau_i & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 1 \\ -\Delta\tau_i & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 0 \\ T_{i-1,k} - \Delta\tau_i & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 0. \end{cases} \tag{5}$$

The minimal up and down times for each unit give limits on $T_{i,k}$. Minimal up time is the time a unit must be on when it has been started. Minimal down time is the time a unit must be off when it has been shot down.

When unit k in time interval i is switched on, i.e. $T_{i-1,k} < 0$ and $u_{i,k} = 1$, a start-up cost $c_{i,k}^{start}$ is defined. This cost is a function of the running time $T_{i-1,k}$,

$$c_{i,k}^{start} = \gamma_k^1 + \gamma_k^2(1 - e^{\gamma_k^3 T_{i-1,k}}). \tag{6}$$

The unit specific parameters γ_k^j are non-negative, i.e. $\gamma_k^j \geq 0, \quad j = 1, 2, 3$, and estimated separately for each unit. Since $T_{i-1,k}$ is by definition negative in the case of start-up, the term $(1-e^{\gamma_k^3 T_{i-1,k}})$ has a value between one and zero. If $|T_{i-1,k}| \gg 0$ the term $(1-e^{\gamma_k^3 T_{i-1,k}})$ is close to one and the resulting start-up cost is almost $\gamma_k^1 + \gamma_k^2$. This situation is called *cold start*. For a *warm start*, when the term $(1 - e^{\gamma_k^3 T_{i-1,k}})$ is close to zero, the cost is approximately γ_k^1. The start-up cost equation (6) has been widely used, see for example Magnusson [Mag91].

The costs and constraints so far presented are common for all types of units. Now define $c_{i,k}(t_{i,k}^f, t_{i,k}^r, q_{i,k}, p_{i,k}, m_{i,k}, u_{i,k})$ as the unit specific production cost and $\Omega_{i,k}(t_{i,k}^f, t_{i,k}^r, q_{i,k}, p_{i,k}, m_{i,k}, u_{i,k})$ as the unit specific constraints for unit k in time interval i. A detailed description of these costs and constraints are given in [DH97].

To make the model as general as possible, introduce a new type of production unit, the *Dummy unit* (with the abbreviation DUM). The Dummy unit is not producing any heat or electric power. Using (1) this restriction is fulfilled defining the constraint

$$t_{i,k}^f = t_{i,k}^r. \tag{7}$$

The production cost for the Dummy unit is equal to zero, i.e.

$$c_{i,k} \equiv 0. \tag{8}$$

When modeling a production plant, including arbitrary number of production units in an arbitrary configuration, the natural is to introduce some sort of network description. Define $G = (N, A)$ as the network which describes the unit configuration. The *directed graph* G consists of a set N of nodes and a set A of arcs. Every single arc $(s, t) \in A$ from node s to node t corresponds to a production unit. Notice that more than one arc can be connected from node s to node t. The number of nodes depends on the plant specific unit configuration, but there are never less then two, since it always exist one input-node s^{in} and one output-node t^{out}. Figure 1 gives a graph describing a plant with five units (arcs) and three nodes.

Define for every node $s \in N \setminus \{s^{in}, t^{out}\}$ in time interval i the mass flow equality constraint

$$\sum_{k \in \widetilde{A}(s)} m_{i,k} = \sum_{k \in A(s)} m_{i,k}, \tag{9}$$

where $\widetilde{A}(s) = \{k : k \text{ corresponds to } (t, s) \in A \text{ and } t \in N\}$, i.e. k corresponds to an incoming arc of s, and $A(s) = \{k : k \text{ corresponds to } (s, t) \in A \text{ and } t \in N\}$, i.e. k corresponds to an outgoing arc of s. Or equally, the mass flow rate into node s equals the mass flow rate out from node s.

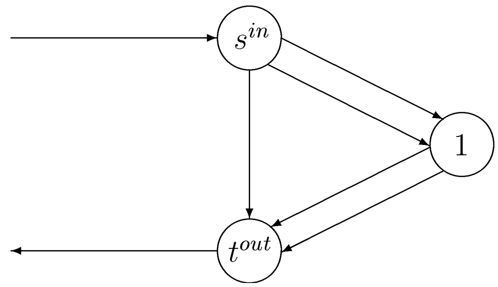

Figure 1: A network describing a plant with five units (arcs) and three nodes.

Constraints relating the input and output temperature for each unit are defined in every single node. A unit directly connected from the return pipe of the plant, i.e. a unit corresponding to an outgoing arc of s^{in}, always have a input temperature $t^r_{i,k}$ equal to the plant return temperature $t^r_{i,res}$. The input temperature for a unit not directly connected to the return pipe of the plant is computed as a mixture of output temperatures from units connected into it. This gives

$$\sum_{k \in \widetilde{A}(s)} t^f_{i,k} m_{i,k} = \sum_{k \in A(s)} t^r_{i,k} m_{i,k} \tag{10}$$

and

$$\begin{cases} t^r_{i,k} = t^r_{i,res} & \text{if } k \in A(s^{in}) \\ t^r_{i,k} = t^r_{i,k'} & \text{if } k \in A(s) \text{ and } k' \in A(s), \end{cases} \tag{11}$$

where $s \neq s^{in}$ and $s \neq t^{out}$.

Solving the short-term production-planning problem over the time horizon $[0, T]$ partitioned into I time intervals, the following assumptions are made for every time interval i:

- The forward temperature demand $t^f_{i,D}$ is known.
- The heat demand $q_{i,D}$ is known.
- The plant return temperature $t^r_{i,res}$ is known.
- The produced net electric power p^{net}_i is sold for the market electricity price $c_{i,e}$.

The flow of the Heat water storage always goes directly from the return pipe of the plant to the forward pipe of the plant. Using the network description, the storage corresponds to an arc from node s^{in} to node t^{out}. Unlike the other units, it is not possible to control the forward temperature $t^f_{i,S}$ for the Heat water storage. Since the storage just is a big tank containing hot water, $t^f_{i,S}$ is given as the storage top temperature $t^{top}_{i,S}$. Defining $t^{bot}_{i,S}$ as the storage bottom temperature, the contribution from the Heat water storage may be directly inserted. This gives the corrected temperature demand as

$$t^{f'}_{i,D} = \begin{cases} \frac{t^f_{i,D} m_{i,D} - t^{top}_{i,S} m_{i,S}}{m_{i,D} - m_{i,S}} & \text{if } m_{i,S} \geq 0 \\ t^f_{i,D} & \text{if } m_{i,S} < 0 \end{cases} \tag{12}$$

and the corrected plant return temperature as

$$t^{r'}_{i,res} = \begin{cases} t^r_{i,res} & \text{if } m_{i,S} \geq 0 \\ \frac{t^r_{i,res} m_{i,D} - t^{bot}_{i,S} m_{i,S}}{m_{i,D} - m_{i,S}} & \text{if } m_{i,S} < 0. \end{cases} \tag{13}$$

Here the mass flow rate demand $m_{i,D}$ is given by (1), i.e. as $m_{i,D} = q_{i,D}/c_p(t^f_{i,D} - t^r_{i,res})$. Deciding the effect on $t^{top}_{i,S}$ and $t^{bot}_{i,S}$ when charging and discharging the storage, i.e. when $m_{i,S} < 0$ and $m_{i,S} > 0$, is a difficult problem not considered in this paper. We assume that the storage temperature has no effect on the rest of the system, i.e. $t^{f'}_{i,D} = t^f_{i,D}$ and $t^{r'}_{i,res} = t^r_{i,res}$ irrespective of the storage mass flow rate $m_{i,S}$ in (12) and (13).

The demand constraints that must be satisfied in time interval i are

$$\sum_{k=1}^{K} q_{i,k} + q_{i,S} \geq q_{i,D} \tag{14}$$

and

$$\frac{\sum\limits_{k \in M} m_{i,k} t^f_{i,k}}{\sum\limits_{k \in M} m_{i,k}} \geq t^f_{i,D}, \tag{15}$$

where K is the number of production units. The set M is defined as all units directly connected to the forward pipe of the plant, i.e. $M = \widetilde{A}(t^{out})$. The Heat water storage is not included in either the set of K units or the set M.

The decision variables for the production units are the binary variable $u_{i,k}$, the output temperature $t^f_{i,k}$, the input temperature $t^r_{i,k}$, the mass flow rate $m_{i,k}$, the heat power production $q_{i,k}$ and electric power production $p_{i,k}$. For the Heat water storage the binary variable $u_{i,S}$, the heat power production $q_{i,S}$ and the energy content $e_{i,S}$ are decision variables.

To summarize, define the short-term production-planning problem as the following mathematical program,

$$\begin{aligned} \min \sum_{i=1}^{I} \sum_{k=1}^{K} & \left(c_{i,k}(t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, u_{i,k}) + c^{start}_{i,k}(u_k)\right) + \sum_{i=1}^{I} c_{i,S}(q_{i,S}, e_{i,S}, u_{i,S}) \\ s.t. \quad & t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, q_{i,S}, e_{i,S} \in \Omega \\ & u_{i,k}, u_{i,S} \in \{0,1\} \\ & \text{minimal up and down times.} \end{aligned} \tag{16}$$

The feasible region Ω is defined by the unit specific constraints $\Omega_{i,k}$ and $\Omega_{i,S}$, the configuration constraints (9), (10), (11) and the demand constraints (14) and (15). The objective function is the total production cost, i.e. the sum of all units' start-up costs and production costs. Notice that no start-up cost is associated with the Heat water storage.

3 Solution Strategy

The solution procedure is based on Lagrangian relaxation, see [Min86]. Relaxing all unit coupling constraints, the short-term production-planning problem (16) decomposes into

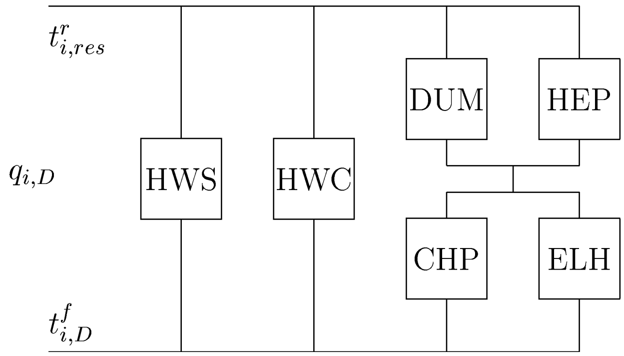

Figure 2: The unit configuration.

one separate problem for each single unit. This methodology was also used by Magnusson in [Mag91] for the solution of the closely related short-term power production-planning problem of an electricity grid. In (16) the unit coupling constraints are the heat demand constraints (14), the forward temperature constraints (15), the mass flow constraints (9) and the temperature constraints (10) and (11). The exact number of relaxed constraints are dependent on the number of time intervals I, on the number of units K and on the unit configuration, i.e. on the number of nodes $|N|$ in the modeled network.

The inequality constraints (14) and (15) are relaxed using the non-negative multipliers λ_i^1 and λ_i^2, $i = 1, ..., I$. The equality constraints (9) and (10) are relaxed using the multipliers $\lambda_i^{3,n}$ and $\lambda_i^{4,n}$, $i = 1, ..., I$ and $n = 1, ..., |N| - 2$. They are unrestricted in sign. The reason why $|N|$ is subtracted by two in the set describing n is because (9) and (10) not are defined in the input-node s^{in} or in the output-node t^{out}. Finally, the equality constraints (11) are relaxed using the unrestricted multipliers $\lambda_i^{5,k}$, $i = 1, ..., I$ and $k = 1, ..., K$.

As an illustrative example, consider the unit configuration in Figure 2. Using the network description gives the directed graph in Figure 1. The sets used in (9) and (10) are $\widetilde{A}(1) = \{DUM, HEP\}$ and $A(1) = \{CHP, ELH\}$. In this case the short-term production-planning problem (16) is

$$\begin{array}{ll}
\min \sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} (c_{i,k}(t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, u_{i,k}) + c^{start}_{i,k}(u_k)) + \sum\limits_{i=1}^{I} c_{i,S}(q_{i,S}, e_{i,S}, u_{i,S}) & \\
s.t. \quad q_{i,hwc} + q_{i,dum} + q_{i,hep} + q_{i,chp} + q_{i,elh} + q_{i,S} \geq q_{i,D} & \\
\qquad m_{i,hwc}t^f_{i,hwc} + m_{i,chp}t^f_{i,chp} + m_{i,elh}t^f_{i,elh} \geq m_{i,hwc}t^f_{i,D} + m_{i,chp}t^f_{i,D} + m_{i,elh}t^f_{i,D} & \\
\qquad m_{i,dum} + m_{i,hep} = m_{i,chp} + m_{i,elh} & \\
\qquad t^f_{i,dum}m_{i,dum} + t^f_{i,hep}m_{i,hep} = t^r_{i,chp}m_{i,chp} + t^r_{i,elh}m_{i,elh} & (17) \\
\qquad t^r_{i,chp} = t^r_{i,elh} & \\
\qquad t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, q_{i,S}, e_{i,S} \in \Omega' & \\
\qquad u_{i,k}, u_{i,S} \in \{0,1\} & \\
\qquad \text{minimal up and down times,} &
\end{array}$$

where all unit coupling constraints are expressed separately. The feasible region Ω' is given as Ω in (16) excluding all unit coupling constraints. The relaxed problem in this

algorithm short-term production-planning
Let $\overline{c} = \infty$ and $\overline{\Phi} = -\infty$.
Initiate lagrange multipliers λ.
while not convergence
  Solve the unit specific problems.
  Generate a primal feasible solution.
  Update $\overline{c}$ and $\overline{\Phi}$.
  if $\overline{c} - \overline{\Phi} \leq \varepsilon$
    convergence
  endif
  Update lagrange multipliers λ.
endwhile

Table 2: The short-term production-planning algorithm.

example becomes

$$\begin{aligned}
\min \sum_{i=1}^{I} \Big(\sum_{k=1}^{K} & (c_{i,k}(t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, u_{i,k}) + c^{start}_{i,k}(u_k)) + c_{i,S}(q_{i,S}, e_{i,S}, u_{i,S}) \\
& +\lambda^1_i (q_{i,D} - q_{i,hwc} - q_{i,dum} - q_{i,hep} - q_{i,chp} - q_{i,elh} - q_{i,S}) \\
& \quad +\lambda^2_i (m_{i,hwc} t^f_{i,D} + m_{i,chp} t^f_{i,D} + m_{i,elh} t^f_{i,D} \\
& \qquad -m_{i,hwc} t^f_{i,hwc} - m_{i,chp} t^f_{i,chp} - m_{i,elh} t^f_{i,elh}) \\
& \quad\quad +\lambda^{3,1}_i (m_{i,chp} + m_{i,elh} - m_{i,dum} - m_{i,hep}) \\
& \qquad\quad +\lambda^{4,1}_i (t^r_{i,chp} m_{i,chp} + t^r_{i,elh} m_{i,elh} - t^f_{i,dum} m_{i,dum} - t^f_{i,hep} m_{i,hep}) \\
& \qquad\qquad +\lambda^{5,chp}_i (t^r_{i,chp} - t^r_{i,elh}) \Big) \\
s.t. \quad & t^f_{i,k}, t^r_{i,k}, q_{i,k}, p_{i,k}, m_{i,k}, q_{i,S}, e_{i,S} \in \Omega' \\
& u_{i,k}, u_{i,S} \in \{0, 1\} \\
& \text{minimal up and down times.}
\end{aligned} \tag{18}$$

The complete algorithm for the solution of the short-term production-planning problem (16) is given in Table 2. In the algorithm $\overline{c}$ and $\overline{\Phi}$ are the best primal and dual solution found so far. The algorithm terminates when the difference $\overline{c} - \overline{\Phi}$ (the duality gap) is small, say less than 0.1 percent of $\overline{c}$. The multipliers λ are determined using a subgradient method with step length given by the Polyak rule II.

Problem (18) separates into one problem for each unit. These unit specific problems are solved using a dynamic programming algorithm, giving a unit commitment feasible with respect to the minimal up and down times. A feasible solution to (16) is then generated using a combination of heuristics and mathematical algorithms. First the unit commitment is corrected with a heuristic method. Given the feasible set of binary variables $u_{i,k}$ and $u_{i,S}$, the economic dispatch problem, i.e. the problem of determining the units production, is solved using a combination of dynamic programming and general-purpose solvers. The solution algorithm for the economic dispatch problem is described in detail in [DH97].

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